Unsteady Supersonic Cascade in Subsonic Axial Flow

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This paper presents an analysis for determining the unsteady flowfield produced by an oscillating cascade placed in a supersonic stream which has a subsonic velocity component normal to the cascade. The analysis is based on the assumptions of an inviscid, two dimensional, linearized flowfield. Solutions for the velocity potential and the blade pressure distributions which satisfy the blade-to-blade periodicity condition are developed explicitly in terms of disturbance functions distributed on blade and wake surfaces. The boundary conditions of flow tangency at blade surfaces and continuity of pressure across wake surfaces provide integral relations which can be solved numerically to evaluate the disturbance functions. Predicted blade pressure distributions are in good agreement with results determined from a previous finite cascade solution. Further, in the two limiting cases of sonic axial velocity and zero frequency, the present solution approaches the lower limit of Lane's solution for supersonic axial flow, and it reduces to an Ackeret type of steady-state solution, respectively. The numerical examples indicate that a single-degree-of-freedom torsional instability will exist over a broad range of cascade parameter values.

Introduction

SUPERSONIC unstalled flutter in the fan stage of modern compressors is one of the most serious problems encountered in the development of advanced high speed jet engines.[1] Since this type of flutter can occur at the design operating condition it imposes a limit on the high-speed operation of the machine. It is therefore important for the compressor designer to have an efficient and accurate mathematical analysis for predicting the onset of flutter and determining the relative influence of the complex array of parameters which control blade response.

Following usual procedure, a compressor stage annulus of differential radial height will be represented as a rectilinear two-dimensional cascade. To operate supersonically, the inlet Mach number relative to the cascade must be greater than one. Two different flow geometries can then occur depending on whether the axial velocity component is supersonic or subsonic. In the first case, referred to as the supersonic leading edge locus condition, no disturbances exist upstream of the blade passages and the unsteady wakes do not influence the flow adjacent to blade surfaces. These features make the supersonic leading edge locus problem amenable to a Laplace transform solution for the unsteady velocity potential in the region upstream of the trailing edge locus of the cascade. Such a solution has been reported by Lane in 1957[2] and provides sufficient information for predicting the aerodynamic forces and moments acting on the blades. Further work on the supersonic axial flow problem was done by Hamamoto,[3] Gorelov,[4] and Platzer and Chalkley.[5] Unfortunately, since modern fans are designed to operate with a subsonic axial velocity, the foregoing analyses cannot be applied to present day engines or those planned for the near future.

When the axial Mach number is less than 1, the cascade is said to have a subsonic leading edge locus (Fig. 1). Unsteady disturbances exist infinitely far upstream of each blade passage and the unsteady wakes influence the flow field adjacent to the lower surfaces of the blades above them. The mathematical difficulties associated with the existence of disturbances upstream of the cascade leading edge locus and the necessity of including the unsteady wakes in the analysis have caused the subsonic leading edge locus problem to remain as one of the significant unsolved problems in the field of aerodynamics. It appears that the first published attempt to solve this problem is due to Gorelov,[4] who outlined a numerical solution procedure based on collocation, but only presented results for the case of supersonic axial flow. Gorelov assumed a normal velocity distribution on the blade wakes which was independent of the blade loading caused by the interference of neighboring blades and wakes. Such an assumption should lead to a violation of the requirement of pressure continuity across wake surfaces. More recently, Kurosaka[6] determined a closed-form result for low-frequency blade motions which is claimed to be accurate to first order

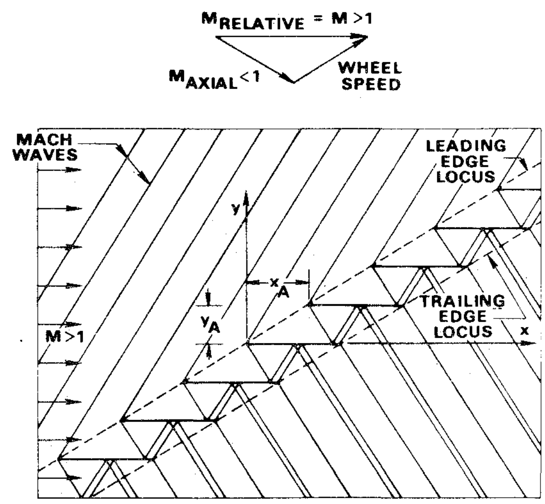

Fig. 1 Supersonic cascade with subsonic leading-edge locus.

Received June 10, 1974; revision received August 9, 1974; presented as Paper 75-22 at the 13th Aerospace Sciences Meeting, Pasadena, Calif., January 20–22, 1975. This research was sponsored by the Pratt & Whitney Aircraft Division of the United Aircraft Corporation.

Index categories: Nonsteady Aerodynamics; Supersonic and Hypersonic Flow; Subsonic and Supersonic Airbreathing Propulsion.

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in reduced frequency. However, some question exists on whether the flowfield upstream of the blade row can be properly represented by the first two terms of a perturbation series based on frequency.[7] Further, the low-frequency regime is of limited concern in the flutter of supersonic fans, since self-excited fan blade oscillations usually occur at frequencies beyond the range of application of the low frequency analysis. Numerical solutions for cascades consisting of a finite number of blades have been obtained by Verdon[8] and by Brix and Platzer.[9] In the finite cascade approximation the number of blades in the cascade must be chosen sufficiently large so that a limiting behavior for the aerodynamic forces and moments can be discerned. It is then assumed that these limit values are representative of the blades of an infinite array. Although this reasoning is intuitively plausible, one weakness of the finite cascade analysis is that mathematical proofs insuring that the response parameters approach limiting values and that these values are representative of infinite cascades have not been determined.[7] Moreover, numerical errors are introduced by difference approximations, and since convergence is generally slow and difference solutions must be computed for each blade and wake passage, the method requires excessive computer time.

Snyder and Commerford[10] have determined that the finite cascade analysis yields predictions which are in agreement with fan rotor experience. A comparison with experimental results for a rotor, which was designed as a supersonic flutter test vehicle, revealed that the flutter speed predicted by the analysis was 80% of the flutter speed of the rotor, and when an amount of mechanical damping typical of material damping was added to the analytical result, the predicted flutter speed was 92% of the measured flutter speed. This correlation is quite good in view of the many simplifying assumptions used in the development of the finite cascade procedure and it thus lends confidence to predictions based on a linearized potential flow model. However, for the reasons cited above, and since calculations must be repeated many times to establish the flutter boundary for a given design, a more accurate and efficient solution of the unsteady supersonic cascade problem is desirable. A solution to the infinite cascade problem which employs closed-form procedures as far as possible and provides the foregoing advantages is described in the present paper. The new solution should also provide a better basis for understanding the physical phenomena associated with unsteady flows past supersonic cascades.

Formulation

In the following discussion all quantities are dimensionless. Lengths have been scaled with respect to blade chord, time with respect to the blade chord divided by the undisturbed freestream speed, and pressure with respect to the freestream density multiplied by one-half the square of the freestream speed. The fluid is assumed to be an inviscid, nonconducting, ideal gas with constant specific heats, and the flow is assumed to be irrotational and isentropic. Disturbances in the supersonic stream are caused by an infinite array of thin, slightly cambered, lifting surfaces or blades which are performing rapid harmonic motions of small amplitude. These motions are generally normal to the blade chord lines which are aligned parallel to the freestream direction. The foregoing conditions lead to a linear boundary value problem for the velocity potential of the disturbed flowfield.[11,12] Linearization provides the additional simplifications that disturbances due to thickness, camber, mean angle of attack, and the unsteady displacement of the blades may be determined separately and the results added to obtain their combined effect, and that boundary conditions on the blade and wake surfaces can be satisfied at the mean positions of these surfaces. Only the unsteady disturbances are of interest here, and consequently the blades are assumed to be flat plates with mean positions at $nx_A \leq x \leq nx_A+1$, $y = ny_A$, $n = 0, \pm 1, \pm 2, \ldots$, where x_A and y_A are the blade stagger and normal gap distances, respectively (Fig. 1). The unsteady wakes are thin vortex sheets which emanate from the trailing edges of the blades and extend infinitely far downstream. Their mean positions are on the lines $1+nx_A < x < \infty$, $y = ny_A$, $n = 0, \pm 1, \pm 2, \ldots$. Consideration will be restricted to the case in which the blades are undergoing identical harmonic motions with arbitrary interblade phase angle, σ, between the motion of adjacent blades. Lane[13] has demonstrated that this apparent limitation leads to a completely general formulation of the flutter problem.

The differential equation governing the disturbed flowfield is

$$\partial^2\psi/\partial y^2 - \mu^2\,\partial^2\psi/\partial x^2 - \mu^2 k^2\psi = 0 \tag{1}$$

where $\mu^2 = M^2-1$, $k = \omega M\mu^{-2}$, and

$$\psi(x,y) = \phi(x,y,t)\exp[i(kMx-\omega t)] \tag{2}$$

Here M is the freestream Mach number, ω is the frequency of the blade motion (reduced frequency based on blade chord), ψ is the modified velocity potential of the unsteady flowfield, and ϕ is the velocity potential as defined in the usual manner. The pressure, p, at a point in the flowfield is given by

$$(p-p_\infty)\exp[i(kMx-\omega t)] = P(x,y) = -2(\partial/\partial x - i\omega\mu^{-2})\psi(x,y) \tag{3}$$

where p_∞ is the freestream pressure and $P(x,y)$ is the modified relative pressure. The cascade geometry and the prescribed form of the blade motion require that the modified potential satisfy the blade-to-blade periodicity condition

$$\psi(x+nx_A, ny_A)\exp(-in\Omega) = \psi(x,y), \quad n = 0, \pm 1, \pm 2, \ldots \tag{4}$$

where $\Omega = \sigma + kMx_A$. It is thus sufficient to determine ψ in a single extended blade passage region, say the zeroth passage defined by $|x| < \infty$, $0 < y < y_A$. The conditions which apply at the upper and lower boundaries of this region are listed below.

1) The modified potential and the fluid normal velocity component are continuous in y along the upstream extensions of the blade chord lines and the normal velocity component is continuous across the blade and wake surfaces. Therefore it follows from Eq. (4) that

$$\psi(x,0^+) = \psi(x,0^-) = \psi(x+x_A, y_A^-)\exp(-i\Omega) \quad x < 0 \tag{5}$$

$$\partial\psi/\partial y(x,0^+) = \partial\psi/\partial y(x,0^-) = \partial\psi/\partial y(x+x_A, y_A^-)\exp(-i\Omega) \quad |x| < \infty \tag{6}$$

2) The flow must be tangent to the blade surfaces and hence

$$\partial\psi/\partial y(x,0) = V(x,0) \quad 0 \leq x \leq 1 \tag{7}$$

where $V(x,0)$, the modified fluid velocity component normal to the zeroth blade surface, is determined from the prescribed blade motion.

3) The pressure must be continuous across wake surfaces and therefore

$$p(x,0^+) = p(x,0^-) = p(x+x_A, y_A^-)\exp(-i\Omega) \quad x > 1 \tag{8}$$

Since conditions on ψ and $\partial\psi/\partial x$ on an upstream boundary of the reference passage (i.e., $x = C_o \ll 0, 0 < y < y_A$) have not been determined, it is not possible to obtain a solution for the modified potential by simply adopting a numerical procedure which considers the reference passage alone. In the analysis which follows, the expression for the modified potential in the reference region is developed in terms of component potentials which account for disturbances originating from different sources. Component potential functions are then superposed to obtain the complete expression for the modified potential. Thus, in addition to the foregoing conditions at the boundaries of the extended blade passage region, the solution for the modified potential must be constructed in such a manner that the following requirements are satisfied: there can be no upstream propagation of disturbances in supersonic flow; unsteady disturbances must be bounded at an infinite distance from their point of origin; disturbance waves impinging on blade surfaces must be reflected; and disturbance waves impinging on wake surfaces must be transmitted through the wake. Mathematical relations involving the modified potential and the blade pressure distributions are more conveniently expressed if the notation

$$B = x_A - \mu y_A \qquad C = 1 - 2\mu y_A \qquad D = x_A + \mu y_A$$

$$I_{m,n}{}^{+}(x-\xi, y) = J_o\{k[(x - mx_A - \xi)^2 - \mu^2(y - ny_A)^2]^{1/2}\} \times U[x - mx_A - \xi + \mu(y - ny_A)]$$

$$I_{m,n}{}^{-}(x-\xi, y) = J_o\{k[(x - mx_A - \xi)^2 - \mu^2(y - ny_A)^2]^{1/2}\} \times U[x - mx_A - \xi - \mu(y - ny_A)]$$

$$K_{m,n}(x-\xi) = [k(x - mx_A - \xi)J_1\{k[(x - mx_A - \xi)^2 - (\mu n y_A)^2]^{1/2}\}/[(x - mx_A - \xi)^2 - (\mu n y_A)^2]^{1/2} + i\omega\mu^{-2} \times J_o\{k[(x - mx_A - \xi)^2 - (\mu n y_A)^2]^{1/2}\}]U(x - mx_A - \xi + \mu n y_A) \tag{9}$$

is adopted. $J_o(x)$ and $J_1(x)$ are the zeroth and first order Bessel functions of the first kind, respectively, and $U(x)$ is the Heaviside or unit step function.

Explicit Expression for the Modified Potential

The unsteady disturbances produced by the blades and their wakes may be represented by the use of disturbance functions, $A_n(x)$, defined on the mean positions of the blade and wake surfaces. An explicit expression for the modified potential in the zeroth or reference extended blade passage region, $0 < y < y_A, |x| < \infty$, can then be obtained in terms of these functions. The unsteady flow in the reference passage results from disturbances produced by the zeroth and first blades of the cascade and their wakes and, in addition, disturbances produced by their neighboring blades and wakes which propagate into the reference passage. The latter include upward propagating disturbances originating below the line $y = 0$ and either upstream of the characteristic $x - \mu y = 0$ or downstream of the characteristic $x - \mu y = 1$, and downward propagating disturbances originating above the line $y = y_A$ and downstream of the characteristic $x + \mu y = 1 + D$.

The potential due to blade and wake disturbances is determined by generalizing Miles' solution for the unsteady flowfield produced by an isolated airfoil oscillating in a supersonic stream.[11] If the mean position of the isolated airfoil and its wake lies on the line $y = 0$, $0 \leqq x < \infty$, then the solution for the modified potential in terms of the upwash on the airfoil and wake surface which satisfies conditions of no upstream propagation of disturbances and boundedness of disturbances at an infinite distance from their source is

$$\mu\psi_I(x, y) = -\mu\psi_I(x, -y) = -\int_o^\infty V(\xi, 0) I_{o,o}{}^{-}(x-\xi, y)\, d\xi \tag{10}$$

The potential due to disturbances produced by the nth blade and wake of the cascade may be represented in a similar manner. In this case the modified normal velocity distribution, $V(x, ny_A)$ on $x \geqq nx_A$, $n = 0, \pm 1, \pm 2, \ldots$, is replaced by the disturbance function distribution $A_n(x)$. The difference between $A_n(x)$ and the modified normal velocity or upwash is due to the unsteady disturbances which exist upstream of the nth blade and its wake, and disturbances generated by the other blades and wakes which pass through the nth wake. The functions $A_n(x)$ are not known a priori, but are determined from relations derived from the boundary conditions on the unsteady flow.

In addition to disturbances generated by the motion of a given blade and its wake, blade surfaces reflect disturbance waves which impinge on them. Expressions for the modified potential due to reflections can be ascertained from Lane's solution to the supersonic leading-edge locus problem.[2] When the freestream axial velocity component is supersonic, the unsteady wakes do not affect the flow adjacent to blade surfaces. Therefore Lane was able to treat the wakes simply as extensions of the blade surfaces and use a Laplace transform solution for the flowfield between two semi-infinite, oscillating flat plates positioned at $y = 0$, $x \geqq 0$, and $y = y_A$, $x \geqq x_A$. The complete potential for this case, which provides a valid cascade solution upstream of the blade trailing-edge locus, is given by

$$\mu\psi(x, y) = \sum_{n=o}^{\infty} \left\{ -\int_o^\infty V(\xi, 0) I_{o,-2n}^{-}(x-\xi, y)\, d\xi - \int_o^\infty V(\xi, 0) I_{o,2n+2}^{+}(x-\xi, y)\, d\xi + \int_{x_A}^\infty V(\xi, y_A) I_{o,2n+1}^{+}(x-\xi, y)\, d\xi + \int_{x_A}^\infty V(\xi, y_A) I_{o,-2n-1}^{-}(x-\xi, y)\, d\xi \right\}, \quad 0 < y < y_A \tag{11}$$

The $n = 0$ term of the first integral in Eq. (11) is the modified potential due to the disturbance wave generated at the upper surface of the plate at $y = 0$. The remaining terms, $n = 1, 2, \ldots$, and the $n = 0, 1, 2, \ldots$, terms of the second integral expression are due to the repeated reflections of this disturbance wave from the lower and upper plates, respectively. The $n = 0$ term of the third integral expression accounts for the disturbance wave originating at the lower surface of the plate at $y = y_A$. The remaining terms of the third and those of the fourth integral expressions account for the repeated reflections of this wave from the upper and lower plates, respectively. Thus, if a solid surface is placed in the stream with mean position at $y = y_A$ along with the airfoil in the previous example, this surface will reflect the disturbance given by Eq. (10) and the potential due to this reflected disturbance is given by the $n = 0$ term of the second integral expression in Eq. (11); i.e.,

$$\mu\psi_R(x, y) = -\int_o^\infty V(\xi) I_{o,2}^{+}(x-\xi, y)\, d\xi \tag{12}$$

The normal velocity at $y = y_A$ due to the incident and reflected waves described by Eqs. (10) and (12) are equal in magnitude but opposite in direction, while the pressures and tangential velocities due to these waves are equal.

With the foregoing ideas in mind, one can construct the solution for the modified potential for the subsonic axial flow problem in the reference extended blade passage region. Two flow configurations which depend on the freestream Mach number and the cascade geometry are of current practical interest (Fig. 2). In the first $D > 1$ and the lower Mach wave from any blade passes behind the lower blades. In the second case $D < 1 < x_A + 3\mu y_A$ and the leading edge Mach waves are reflected once by the adjacent blades below. The points labeled 1, 2, and 3 in Fig. 2 denote the locations at which Mach waves impinge on and are reflected by the zeroth blade. Discontinuities occur in the zeroth blade pressure distributions at these points. The solution procedure is better illustrated by a development leading to an expression for the modified potential for the second case. The result for the simpler flow geometry is then readily obtained by neglecting the terms

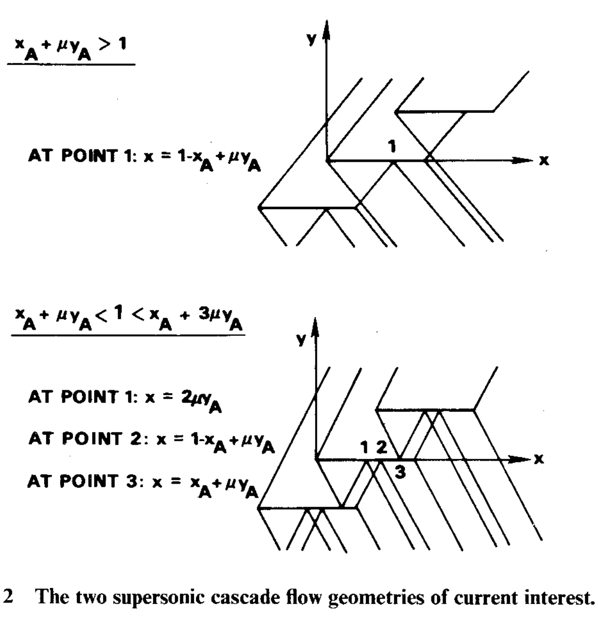

Fig. 2 The two supersonic cascade flow geometries of current interest.

representing the additional reflections. More complicated flow geometries can be treated by a straightforward extension of the concepts presented below; however, such cases are not of interest for present applications.

Disturbances upstream of the characteristic $x-\mu y=0$ originate at the leading edge segments, $nx_A \leqq x < nx_A+B$, $y = ny_A$, of the blades below the reference passage. The potential, $\psi_1(x, y)$, due to these disturbances is obtained by generalizing Miles' isolated airfoil result, Eq. (10), and is given by

$$\mu\psi_1(x,y) = -\sum_{n=-\infty}^{-1}\int_{nx_A}^{nx_A+B} A_n(\xi)I^-_{o,n}(x-\xi,y)\,d\xi \quad 0 \leqq y \leqq y_A \tag{13}$$

Disturbances originating at the nth blade surface ($n < 0$) downstream of the point nx_A+B do not propagate directly into the reference passage since they are reflected by the blades immediately above them.

Disturbances produced by the zeroth blade and its wake are accounted for in the same manner; however, those originating on the interval $B \leqq x \leqq 1+B$, $y = 0$ impinge on the first blade giving rise to a reflected disturbance which also contributes to the unsteady flowfield in the reference passage. When $D < 1 < x_A+3\mu y_A$ there will be further reflections downstream and the modified potential, $\psi_2(x, y)$, due to this disturbance wave system has the form

$$\mu\psi_2(x,y) = -\int_o^\infty A_o(\xi)I^-_{o,o}(x-\xi,y)\,d\xi - \int_B^{1+B} A_o(\xi)I^+_{o,2}(x-\xi,y)\,d\xi - \int_B^C A_o(\xi)I^-_{o,-2}(x-\xi,y)\,d\xi - \int_B^C A_o(\xi)I^+_{o,4}(x-\xi,y)\,d\xi \quad 0 < y < y_A \tag{14}$$

The first integral term in Eq. (14) accounts for disturbances originating at the zeroth blade and wake. The second term is due to the reflection of this disturbance by the first blade. These terms have the same form as Eqs. (10) and (12), respectively. The third and fourth terms follow from Lane's solution, Eq. (11), and account for the subsequent reflections of the original disturbance wave by the zeroth and first blade. The upper limit on the integral signs in the terms representing reflected disturbances are finite because the reflecting blade surfaces have finite length and the blade wakes have been assumed to transmit impinging disturbance waves.

Similarly the modified potential, $\psi_3(x, y)$, due to disturbance waves originating at the first blade and its wake and their reflections off the zeroth and first blade is given by

$$\mu\psi_3(x,y) = \int_{x_A}^{\infty} A_1(\xi)I^+_{o,1}(x-\xi,y)\,d\xi + \int_{x_A}^{1-\mu y_A} A_1(\xi)I^-_{o,-1}(x-\xi,y)\,d\xi + \int_{x_A}^{1-\mu y_A} A_1(\xi)I^+_{o,3}(x-\xi,y)\,d\xi, \quad 0 < y < y_A \tag{15}$$

The potential component $\psi_4(x, y)$ accounts for disturbances produced by the blade wakes below the first blade which propagate upward, and the reflections from the lower surface of the first blade of those disturbances produced at the wake segments $1+n\mu y_A < x \leqq 1+B+n\mu y_A$, $y = ny_A$, $n = -1, -2, \ldots$. It follows that

$$\mu\psi_4(x,y) = -\sum_{n=-\infty}^{-1}\int_{1+\mu n y_A}^{\infty} A_n(\xi)I^-_{o,n}(x-\xi,y)\,d\xi - \sum_{n=-\infty}^{-1}\int_{1+\mu n y_A}^{1+B+\mu n y_A} A_n(\xi)I^+_{o,2-n}(x-\xi,y)\,d\xi \quad 0 < y < y_A \tag{16}$$

The first term in Eq. (16) accounts for the disturbances generated at the wakes below the first blade and the second term represents the reflections of these wake disturbances by the first blade.

Finally, the modified potential component, $\psi_5(x, y)$ is due to disturbances originating at the blade and wake surfaces above the first blade which propagate downward and are not reflected upward by lower blades, and in addition, to the reflection of upward propagating disturbances by the blades above the first blade, which are not intercepted by the blades below. The expression for $\psi_5(x, y)$ for the example flow geometry has the form

$$\mu\psi_5(x,y) = \sum_{n=2}^{\infty}\int_{1+nx_A-D}^{\infty} A_n(\xi)I^+_{o,n}(x-\xi,y)\,d\xi + \sum_{n=2}^{\infty}\int_{nx_A}^{1+nx_A-D} A_n(\xi)I^+_{o,n+2}(x-\xi,y)\,d\xi - \sum_{n=1}^{\infty}\int_{nx_A+C}^{1+nx_A+B} A_n(\xi)I^+_{o,n+2}(x-\xi,y)\,d\xi - \sum_{n=1}^{\infty}\int_{nx_A+B}^{nx_A+C} A_n(\xi)I^+_{o,n+4}(x-\xi,y)\,d\xi - \sum_{N=2}^{\infty}\sum_{n=-\infty}^{N-2}\int_{1+(N-1)B+\mu n y_A}^{1+NB+\mu n y_A} A_n(\xi)I^+_{o,2N-n}(x-\xi,y)\,d\xi \quad 0 < y < y_A \tag{17}$$

This component of the modified potential accounts for disturbances which have no influence on the zeroth and first blade pressure distributions. The first summation term in Eq. (17) represents disturbances originating at the upper blades and their wakes and the remaining terms account for the reflections by the upper blades of disturbances which impinge on them from below.

The modified potential distribution in the reference passage region is obtained by a summation of the foregoing component terms, i.e.,

$$\psi(x,y) = \sum_{i=1}^{5}\psi_i(x,y) \qquad 0 < y < y_A \tag{18}$$

The cascade geometry and the prescribed form of the blade motion suggest that the blade disturbance functions satisfy the following blade-to-blade periodicity relations

$$A_n(x+nx_A)e^{-in\Omega} = A_o(x) = A(x) \qquad n = 0, \pm 1, \pm 2, \ldots \tag{19}$$

After some algebra and the assumption that the operations of summation and integration can be interchanged, it follows from Eqs. (13–19) that the modified potential distribution in the reference passage for the example flow geometry is given by

$$\mu\psi(x,y) = -\int_o^B A(\xi)\sum_{n=-\infty}^{-1} e^{in\Omega}I^-_{n,n}(x-\xi,y)\,d\xi - \int_o^\infty A(\xi)I^-_{o,o}(x-\xi,y)\,d\xi - \sum_{n=-\infty}^{-1}\int_{1-nB}^{\infty} A(\xi)e^{in\Omega}I^-_{n,n}(x-\xi,y)\,d\xi - \int_B^C A(\xi)I^-_{o,-2}(x-\xi,y)\,d\xi - \int_B^1 A(\xi)I^+_{o,2}(x-\xi,y)\,d\xi - \int_C^1 A(\xi)\sum_{n=1}^{\infty} e^{in\Omega}I^+_{n,n+2}(x-\xi,y)\,d\xi - \int_B^C A(\xi)\sum_{n=0}^{\infty} e^{in\Omega}I^+_{n,n+4}(x-\xi,y)\,d\xi + \int_o^\infty A(\xi)e^{i\Omega}I^+_{1,1}(x-\xi,y)\,d\xi + \int_{1-D}^{\infty} A(\xi)\sum_{n=2}^{\infty} e^{in\Omega}I^+_{n,n}(x-\xi,y)\,d\xi + \int_o^{1-D} A(\xi)\sum_{n=1}^{\infty} e^{in\Omega}I^+_{n,n+2}(x-\xi,y)\,d\xi + \int_o^{1-D} A(\xi)e^{i\Omega}I^-_{1,-1}(x-\xi,y)\,d\xi - \sum_{N=1}^{\infty}\sum_{n=-\infty}^{N-1}\int_{1+(N-n-1)B}^{1+(N-n)B} A(\xi)e^{in\Omega}I^+_{n,2N-n}(x-\xi,y)\,d\xi, \quad 0 < y < y_A \tag{20}$$

It may be readily verified, though the algebra is tedious, that the expression for the potential given by Eq. (20) satisfies the differential equation governing the unsteady flowfield, Eq. (1), and the blade-to-blade periodicity conditions, Eqs. (5) and (6). The modified pressure distributions on the upper and lower surfaces of the zeroth or reference blade follow from Eqs. (3) and (20) and are given by

$$\mu P(x,0^+)/2 = -\int_o^B A(\xi)\sum_{n=-\infty}^{-1} e^{in\Omega}K_{n,n}(x-\xi)\,d\xi + A(x) - \int_o^1 A(\xi)K_{o,o}(x-\xi)\,d\xi + 2U(x-D)[A(x-2\mu y_A)-A(x-D)e^{i\Omega}] - 2\int_B^C A(\xi)K_{o,-2}(x-\xi)\,d\xi + 2\int_o^{1-D} A(\xi)e^{i\Omega}K_{1,-1}(x-\xi)\,d\xi \qquad 0\leq x\leq 1 \tag{21}$$

and

$$\mu P(x,0^-)/2 = \mu P(x+x_A, y_A{}^-)e^{-i\Omega}/2 = -\int_o^B A(\xi)\sum_{n=-\infty}^{-1} e^{in\Omega}K_{n,n}(x-\xi)\,d\xi - A(x) + \int_o^1 A(\xi)K_{o,o}(x-\xi)\,d\xi + 2[U(x)-U(x+B-1)]A(x+B)e^{-i\Omega} - 2\int_B^1 A(\xi)e^{-i\Omega}K_{-1,-1}(x-\xi)\,d\xi + 2U(x+B-1)\sum_{n=-\infty}^{-1}\left[A(x-nB)e^{in\Omega} - \int_{1-(n+1)B}^{1-nB} A(\xi)e^{in\Omega}K_{n,n}(x-\xi)\,d\xi\right] + 2[U(x-2\mu y_A)-U(x+B-1)]\times[A(x+x_A-3\mu y_A)e^{-i\Omega}-A(x-2\mu y_A)] - 2\int_B^C A(\xi)e^{-i\Omega}K_{-1,-3}(x-\xi)\,d\xi + 2\int_o^{1-D} A(\xi)K_{o,-2}(x-\xi)\,d\xi \qquad 0\leq x\leq 1 \tag{22}$$

The first integral term in Eqs. (21) and (22) gives the contribution to the reference blade pressure distributions from disturbances generated at the leading edge segments, $nx_A \leq x < nx_A+B$, $n = -1, -2, \ldots$, of the blades below it. The second and third terms in these equations represent the contribution from disturbances produced by the reference blade. The second infinite series term in Eq. (22) is the contribution from disturbances generated by the lower wakes and their reflection from the lower surface of the reference blade. Finally, the remaining terms in the two pressure expressions represent the contributions due to disturbances generated by the blades adjacent to the reference blade and the aerodynamic interactions between these two blades and the reference blade.

Expressions for the modified potential and blade pressure distributions for flows in which trailing edge Mach waves pass behind the blades below ($D > 1$) can be readily obtained from Eqs. (20–22). The modified potential for such cases follows after eliminating the fourth, seventh, tenth, and eleventh integral terms from Eq. (20) and setting the lower limits on the sixth and ninth integrals equal to B and 0, respectively. The reference blade pressure distributions are obtained by eliminating the terms containing the factors $U(x-D)$ and $[U(x-2\mu y_A)-U(x+B-1)]$ from Eqs. (21) and (22), respectively, and also the last two integral terms in both of these equations.

Disturbance Function Relations

To evaluate the modified potential in the reference passage and the reference blade pressure distributions, values of the disturbance function, $A(x)$, must be determined by using the boundary conditions of flow tangency at the upper surface of the zeroth blade and continuity of pressure across the zeroth wake. After differentiating Eq. (20) with respect to y, setting $y = 0$, and substituting the result into the flow tangency condition, Eq. (7), it follows that

$$A(x) = V(x,0) - \int_o^B A(\xi)K(x-\xi)\,d\xi, \qquad 0\leq x\leq 1 \tag{23}$$

where

$$K(x) = k\mu y_A \sum_{n=-\infty}^{-1} n\,e^{in\Omega}\frac{J_1\{k[(x-nx_A)^2-(\mu n y_A)^2]^{1/2}\}}{[(x-nx_A)^2-(\mu n y_A)^2]^{1/2}} \tag{24}$$

Recall that $V(x,0)$ is a prescribed quantity in the interval $0\leq x\leq 1$. The integral term in Eq. (23) represents the influence of disturbances generated at the leading edge segments, $nx_A \leq x < nx_A+B$, $n = -1, -2, \ldots$, of the infinite array of blades in the negative y-direction on the unsteady disturbances generated by the motion of the zeroth blade. For $0\leq x\leq B$, Eq. (23) is a Fredholm integral equation with unknown function $A(x)$. The solution of this equation can be obtained by iteration; e.g., set $A_{(1)}(x) = V(x,0)$ and evaluate

$$A_{(m+1)}(x) = V(x,0) - \int_o^B A_{(m)}(\xi)K(x-\xi)\,d\xi, \qquad m = 1,2,3,\ldots \tag{25}$$

until

$$|A_{(m+1)}(x) - A_{(m)}(x)| < \varepsilon \qquad 0 < x \leq B \tag{26}$$

where ε is a preselected small parameter. A sufficient condition for convergence of this iterative procedure is[14]

$$\max_{x\in[0,B]}\int_o^B |K(x-\xi)|\,d\xi < 1 \tag{27}$$

Once $A(x)$ has been determined in the interval $0\leq x\leq B$ its its values in the range $B < x \leq 1$ can be obtained from Eqs. (23) and (24) by straightforward integration.

Values of the disturbance function, $A(x)$, on the zeroth or reference blade, $0\leq x\leq 1$, provide sufficient information for determining the modified potential in the reference blade passage up to the trailing-edge Mach wave of the zeroth blade, $x = 1+\mu y$, and hence, the pressure distributions on the entire upper surface of the zeroth blade and on the lower surface of the first blade from its leading edge to the point of impingement, $x = 1+\mu y_A$, of the zeroth blade trailing-edge Mach wave. The lower surface pressure on the zeroth blade in the interval $0\leq x\leq 1-B$ follows from the foregoing result for the first blade and the blade-to-blade periodicity condition. To complete the solution for the potential in the reference passage and the pressure distribution on the lower surface of the zeroth blade the disturbance function, $A(x)$, must be determined on the reference wake ($1 < x < \infty$).

It follows from Eqs. (3), (8), and (20) that pressure will be continuous across the reference wake if the disturbance function satisfies the relation

$$A(x) - \int_1^x A(\xi)K_{o,o}(x-\xi)\,d\xi = F(x) - G[A(x),x], \qquad x > 1 \tag{28}$$

where

$$F(x) = \int_o^1 A(\xi)K_{o,o}(x-\xi)\,d\xi - \int_B^1 A(\xi)e^{-i\Omega}K_{-1,-1}(x-\xi)\,d\xi + \int_B^C A(\xi)[K_{o,-2}(x-\xi)-e^{-i\Omega}K_{-1,-3}(x-\xi)]\,d\xi + \int_o^{1-D} A(\xi)[K_{o,-2}(x-\xi)-e^{i\Omega}K_{1,-1}(x-\xi)]\,d\xi \qquad x > 1 \tag{29}$$

and

$$G[A(x),x] = \int_{1-B}^{1} \sum_{n=-\infty}^{-1} e^{in\Omega} A(\xi - nB) K_{o,n}(x - \xi - \mu n y_A)\, d\xi \qquad x > 1 \qquad (30)$$

For the case $D > 1$ the last two integral terms do not appear in Eq. (29). Equations (28–30) define the disturbance function distribution which is required to maintain pressure continuity across the blade wakes, or equivalently, which is required to conserve the circulation in the unsteady flowfield. The disturbance function is linearly related to the vorticity distribution and hence values of $A(x)$ on the wake depend on the aerodynamic loading, represented by the right-hand side of Eq. (28), on the blade preceding it. The function $F(x)$ results from unsteady disturbances generated by the motion of the reference blade and from disturbances generated at the surface of adjacent blades which impinge on the reference blade. This function depends on values of the disturbance function, $A(x)$, for $0 \leqq x \leqq 1$, and hence can be regarded as a known function in Eq. (28). $G[A(x),x]$ arises from disturbances generated at the lower wakes which impinge on and are reflected by the lower surface of the reference blade. This term depends on values of $A(x)$ for $1 < x < \infty$ and thus is an unknown quantity in the wake integral equation. If $G[A(x),x]$ could be neglected, Eq. (28) would reduce to a Volterra integral equation of the second kind, which would admit a simple finite difference solution[15] for the disturbance function distribution on the reference wake. The term due to the wakes below the reference blade introduces the complication that the value of the disturbance function at a given wake location, say $x = x_0$, depends on its values downstream of this location; i.e., $x_0 < x < \infty$. A result similar to Eq. (28) has been previously derived for the isolated airfoil.[16] In this case $G[A(x),x] = 0$, the interference terms do not appear in the expression for $F(x)$, and $A(x) = V(x,0)$.

The numerical results reported in the following section have been obtained by using the following iterative scheme to solve the wake integral equation. Successive estimates, $A_{(m+1)}(x)$, for the disturbance function were determined by finite difference solutions of the integral equations

$$A_{(m+1)}(x) - \int_{1}^{x} A_{(m+1)}(\xi) K_{o,o}(x-\xi)\, d\xi = F(x) - G[A_{(m)}(x), x] \qquad m = 0,1,2,\ldots \qquad (31)$$

where $A_{(o)}(x) = 0$. The iteration process continues until

$$|A_{(m+1)}(x) - A_{(m)}(x)| < \varepsilon \qquad \text{for} \qquad 1 < x < L \qquad (32)$$

and then $A(x)$ is set equal to $A_{(m+1)}(x)$. As before ε is a preselected small parameter. The distance L must be chosen large enough so that an accurate estimate for the sums of the infinite series in Eqs. (22) and (30) can be obtained.

To obtain the numerical results described in this paper the various integral expressions have been evaluated using finite difference approximations, and the sums of the infinite series in Eqs. (21), (22), (24), and (30) have been evaluated numerically. The partial sums of these series tend to oscillate about apparent limiting values, but convergence is rather slow. This behavior is similar to that exhibited by the numerical results for the response parameters in the earlier finite cascade solution.[8] The sum of each series has been assumed to be equal to the mean value of its partial sums which is determined by averaging the partial sums over a selected number of cycles of oscillation. Unfortunately, convergence of the various series has not been proved.

Comparison with Previous Solutions and Numerical Examples

To indicate the validity of the foregoing solution, its behavior in the limiting cases $B \to 0$ (sonic leading edge locus) and $k \to 0$ (steady flow) will be examined. Further, a comparison will be made between blade pressure distributions predicted by the present analysis and the finite cascade analysis described in Ref. 8. When $B = 0(x_A = \mu y_A)$ there are no disturbances upstream of the blade row and unsteady wake disturbances do not affect the flow adjacent to blade surfaces. In this case the present solution should agree with the lowest limit of Lane's solution to the supersonic leading edge locus problem. It follows from Eqs. (21) and (22) that the terms which represent the influence of the upstream disturbances and the influence of the wake disturbances and their reflections from the lower surface of the zeroth blade on the blade pressure distributions are equal to zero. It is readily observed from Eq. (23) that the disturbance function is equal to the modified normal velocity on blade surfaces. When this result is substituted into Eqs. (21) and (22) the expressions for the blade pressure distributions become identical to those given by Lane's analysis[2] for flow configurations defined by $D < 1 < x_A + 3\mu y_A$. In addition, the term $G[A(x),x]$ in the wake integral equation, Eq. (28), vanishes and the resulting Volterra integral equation admits a simple finite difference solution[15] for the wake disturbance function distribution. Thus, the method of solution described in the present paper can be used to extend Lane's supersonic axial flow result into the region downstream of the blade row. Such an extension might be of interest in future studies on noise generation.

It follows from Eqs. (9, 23, 24, and 28–30) that in the steady flow limit ($k = 0$)

$$A(x) = V(x)[U(x) - U(x-1)] \qquad (33)$$

i.e., the disturbance function is equal to the modified normal velocity on blade surfaces and it is equal to zero on wake surfaces. In addition, if the blades are undergoing pitching motions of amplitude α, where α is positive for clockwise blade rotations, then in the limit as k and σ approach zero, $V(x,0) = -\alpha$ and it follows from Eqs. (21) and (22) that

$$p(x,0^-) - p(x,0^+) = 4\mu^{-1}\alpha U(x-1+B), \qquad 0 \leqq x \leqq 1 \qquad (34)$$

as expected. Equation (34) indicates that there is no pressure difference across the blade upstream of the point of impingement of the trailing edge Mach wave from the blade below, and that downstream of this point the blade behaves as an isolated airfoil.

Although the agreement between the present solution and previous analytic results in the foregoing limiting cases is encouraging, it does not guarantee the validity of the new procedure. Neither limiting case contains the effects of unsteady disturbances upstream of the blade row nor the effects of unsteady wake disturbances on blade pressure distributions. The representation of these disturbances in terms of infinite series for which convergence has not been proved raises the major questions concerning the validity of the new procedure. A comparison of the results predicted by the present analysis and the finite cascade analysis is discussed below. The finite cascade analysis also suffers from the deficiency of lacking a convergence proof; however, agreement between results predicted by two basically different procedures would add further confidence in the analytical predictions. As mentioned earlier, good correlations have been achieved between finite cascade predictions and fan rotor measurements.[10]

In the numerical examples which follow, cascade parameters have been chosen to illustrate physical behavior for the two flow configurations which have been discussed throughout this paper. For cascade A, with $x_A = 0.68$, $y_A = 0.4$, and $\mu = 0.9$ ($s = 1.268$, $\theta = 30.5°$, $M = 1.345$) the leading-edge Mach waves pass behind lower blades. For cascade B, with $x_A = 0.6$, $y_A = 0.3$, and $\mu = 0.8$ ($s = 1.490$, $\theta = 26.6°$, $M = 1.281$) leading-edge Mach waves are reflected once by the adjacent blades below. Here $s = (x_A{}^2 + y_A{}^2)^{-1/2}$ and $\theta = \tan^{-1}(y_A/x_A)$ denote the cascade solidity and stagger angle, respectively. The pressure distribution on the lower surface of the reference blade of cascade A is discontinuous only at the point of impingement of the trailing-edge Mach wave from the adjacent blade below ($x = 1-B$, $y = 0$), and the upper surface pressure distribution is continuous. In addition to the pressure discontinuity cited previously, the upper surface pressure distribution on the reference blade of cascade B

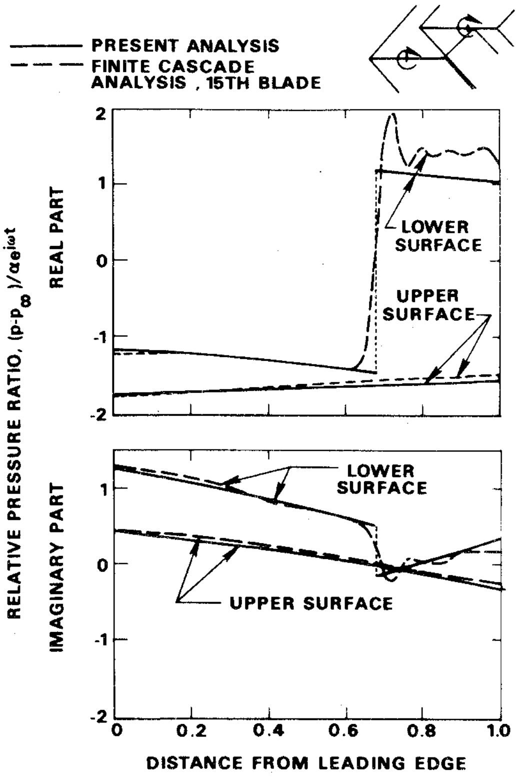

Fig. 3 Comparison of predicted blade pressure distributions for pitching motion of cascade A: $k = 1.0$, $\sigma = 0$, $x_n = 0.5$.

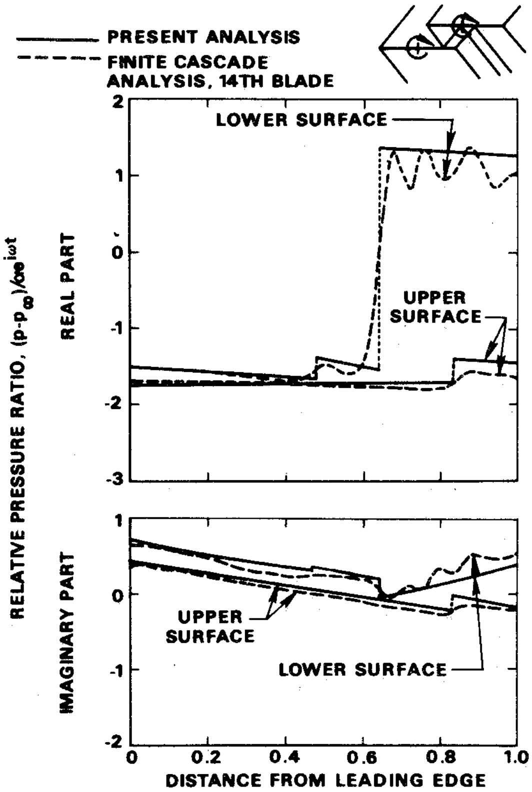

Fig. 4 Comparison of predicted blade pressure distributions for pitching motion of cascade B: $k = 1.0$, $\sigma = 0$, $x_n = 0.5$.

is discontinuous at the point $x = D, y = 0$ where the leading-edge Mach wave from the adjacent upper blade strikes the reference blade, and the lower surface pressure distribution is discontinuous at the point $x = 2\mu y_A$, $y = 0$ where the reflection of the reference blade leading-edge Mach wave from the blade below strikes the reference blade.

For single-degree-of-freedom pitching (torsional) motions, the normal velocity, $v(x, y, t)$, at the blade surfaces is given by

$$v(x+nx_A, ny_A, t) = V(x+nx_A, ny_A)\exp\{i[\omega t - kM(x+nx_A)]\} = -[\alpha + i\omega\alpha(x-nx_A-x_n)]\exp[i(\omega t+n\sigma)],$$
$$0 \leq x \leq 1, \qquad n = 0, \pm 1, \pm 2, \ldots \tag{35}$$

where α is the amplitude of the pitching displacement ($\alpha e^{i\omega t}$) and x_n is the distance from the leading edge of each blade to its axis of rotation. The angular displacement is assumed to be positive for clockwise rotations of the blade. Relative pressure distributions predicted with the finite cascade analysis[8] and the present analysis are shown in Figs. 3 and 4. The real part of the ratio $(p-p_\infty)/\alpha e^{i\omega t}$ represents the relative pressure per unit amplitude when the blade has reached its maximum angular displacement in the clockwise direction, while the imaginary part is the relative pressure per unit amplitude when the blade is in its mean position and rotating clockwise. It can be seen from the figures that close agreement has been obtained between pressure distributions predicted by the two procedures. Differences can be primarily attributed to the finite cascade results, given in Figs. 3 and 4, not being limiting values, but merely the calculated values of the relative pressures acting on the 15th blade for configuration A or the 14th blade for configuration B. In addition, the finite difference marching procedure used in Ref. 8 does not recognize pressure discontinuities as such, but only approximates them by sharp changes in the pressure distribution. This numerical approximation also gives rise to the ripples which appear in the predicted pressure distributions downstream of discontinuities.

The effects of aerodynamic interactions are revealed by a comparison of the pressure difference distributions acting on the blades of an infinite cascade with those acting on isolating airfoils operating under the same conditions. Results for single-degree-of-freedom, in-phase and out-of-phase, pitching motions for the two cascade configurations, A and B, and the isolated airfoil are shown in Figs. 5 and 6. The significant difference between pressure distributions acting on the isolated airfoil and the reference blades of cascades A and B are primarily due to

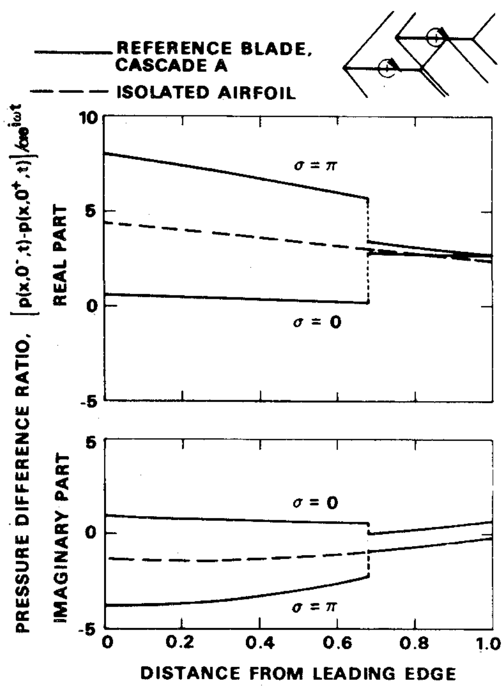

Fig. 5 Pressure difference distributions for pitching motions of cascade A and isolated airfoil: $k = 1.0$, $x_n = 0.5$.

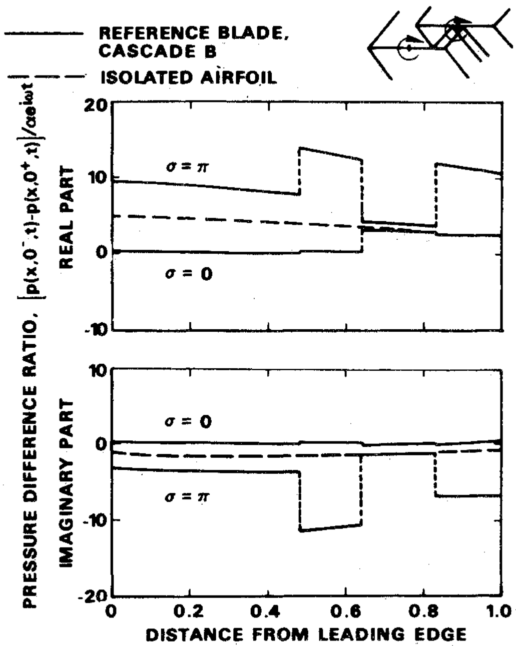

Fig. 6 Pressure difference distributions for pitching motions of cascade B and isolated airfoil: $k = 1.0$, $x_n = 0.5$.

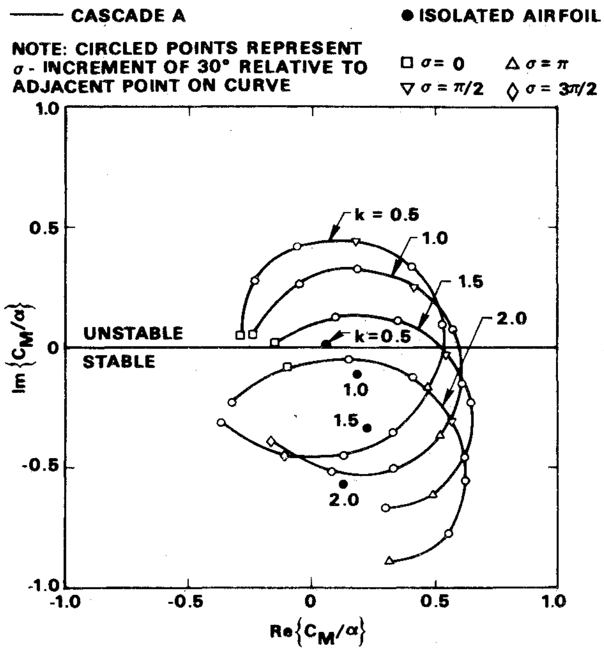

Fig. 7 Moment coefficients due to pitching motions of cascade A and isolated airfoil.

the disturbance waves caused by the presence of the two blades adjacent to the reference blade. Upstream disturbances generated at the lower blade leading-edge segments, $nx_A \leq x \leq nx_A + B$, $y = ny_A$, $n = 1, -2, \ldots$, and those generated at the lower wakes appear to play a less important role. This can be seen by examining the results for the in-phase pitching of the blades in cascade A. Isolated airfoil pressure distributions satisfy the relation[16]

$$p(x,0^-,t) - p(x,0^+,t) = 2p(x,0^-,t) = -2p(x,0^+,t) \tag{36}$$

The curves in Figs. 3 and 5 reveal that there are substantial differences between isolated airfoil and reference blade lower surface pressures upstream of the point, $x = 1 - B$, where the trailing-edge Mach wave from the adjacent lower blade of the cascade strikes the reference blade. Lower surface pressures downstream of this point and upper surface pressures acting on the reference blade no not differ as greatly from those acting on the isolated airfoil. Disturbances generated at the upper surface of the blade below the reference blade impinge on the segment $0 \leq x \leq 1 - B$ of the reference blade and cause the substantial differences between isolated airfoil and reference blade lower surface pressures. Downstream of the point $x = 1 - B$ unsteady wake disturbances impinge on the reference blade.

Discontinuities in the pressure difference distribution acting on the reference blade of cascade B occur at $x = 2\mu y_A$, $x = 1 - B$, and $x = D$, Fig. 6. Upstream of the first discontinuity disturbances generated by the adjacent lower blade impinge on the lower surface of the reference blade. In the interval between the first and second discontinuities, three blade-generated disturbance waves impinge on the lower surface of the reference blade. These consist of disturbances caused by the motion of the lower blade, the subsequent reflection of these disturbances by the lower blade, and the reflection from the lower blade of disturbances originating at the lower surface of the reference blade. Reference blade lower surface pressures upstream of the second discontinuity differ substantially from those occurring on the isolated airfoil. Downstream of this location the flow adjacent to the reference blade is influenced by the lower wakes and the lower surface pressures are in closer agreement with isolated airfoil pressures (Figs. 4 and 6). Downstream of the third discontinuity disturbances generated by the blade above the reference blade impinge on the upper surface reference blade.

For single-degree-of-freedom pitching motions the work done by the airstream over one cycle of blade motion is[17,18]

$$W_{\text{per cycle}} = \pi\omega\alpha I_m\{C_M\} \tag{37}$$

where the moment coefficients are given by

$$C_M e^{i\omega t} = -\int_0^1 (x - x_n)[p(x,0^-) - p(x,0^+)]\,dx \tag{38}$$

When the work per cycle is positive ($Im\{C_M\} > 0$) the blade motion receives energy from the airstream and this motion will be unstable. Interference effects on the stability of airfoil motion are dramatically illustrated by the moment coefficient diagrams shown in Figs. 7 and 8. The possibility of unstable,

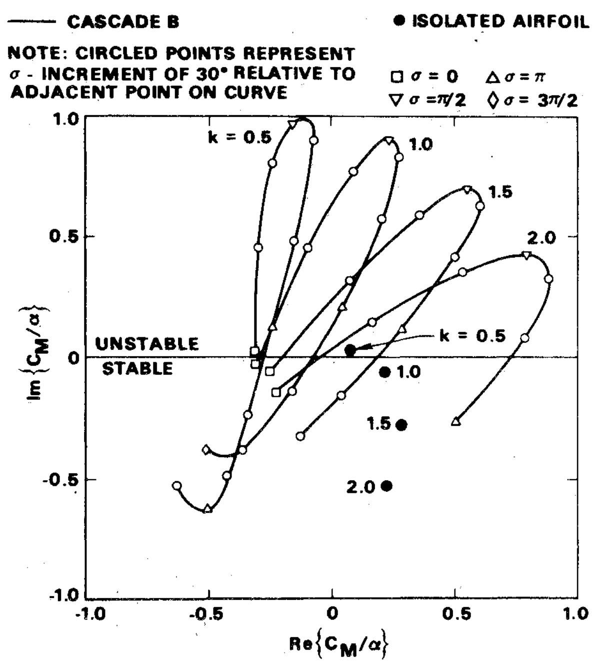

Fig. 8 Moment coefficients due to pitching motions of cascade B and isolated airfoil.

low-frequency, isolated airfoil pitching oscillations has been reported earlier by Garrick and Rubinow[19] and by Miles.[11] Present calculations indicate that the pitching motions of an isolated airfoil will be unstable at $k = 0.5$ when $M = 1.345$ (Fig. 7) and $M = 1.281$ (Fig. 8), and that such motions will become stable as the compressible reduced frequency, $k = \omega M \mu^{-2}$ is increased. The numerical results, plotted in Figs. 7 and 8, confirm earlier indications[1,6,8,10] that unstable cascade pitching motions will occur over a broad range of frequencies and interblade phase angles. Again stability appears to be enhanced as k increases. These results emphasize the serious influence of aerodynamic interactions on the response of blades in cascade. The stability of plunging (bending) motions is governed by the sign of the imaginary part of the lift coefficient. Present calculations have not revealed a plunging instability for either the isolated airfoil or the blades in cascade. The moment coefficient loops plotted in Figs. 7 and 8 are not closed due to the failure of the wake iteration procedure outlined in Eqs. (31) and (32) to converge for certain combinations of the cascade parameters. The range in which this iteration procedure appears to diverge is given approximately by

$$|\sigma + kMx_A| \leqq k(x_A^2 - \mu^2 y_A^2)^{1/2} \tag{39}$$

Conclusions

A solution for the unsteady velocity potential and the blade pressure distributions generated by the small-amplitude harmonic motions of a cascade of airfoils placed in a supersonic stream which has a subsonic axial velocity component normal to the cascade has been determined. This solution should provide a basic design tool for establishing the flutter boundary of proposed supersonic fans. It will permit the compressor designer to predict the aerodynamic forces and moments acting on the blades, and hence to investigate the stability of the blade motion for given values of the freestream Mach number, the cascade solidity and stagger angle, and the amplitude, reduced frequency, interblade phase angle, and mode of blade motion.

The finite cascade solution procedures developed by Verdon[8] and by Brix and Platzer[9] require that numerical results be computed in each blade and wake passage. Since response parameters converge slowly to limiting values many passages must be considered, and therefore the finite cascade analysis requires substantial computing time. In the present analysis the solution for the modified potential is developed for a single passage (reference passage). The influence of neighboring blades and wakes on the flow in the reference passage is represented by infinite series expressions whose sums can be evaluated relatively quickly. Hence, this approach is inherently more efficient than finite cascade procedures. Computational efficiency is especially important since many parametric combinations must be analyzed to determine the flutter boundary for a given supersonic fan.

Calculations have indicated that plunging (bending) motions are stable and that pitching (torsional) motions are unstable (according to linear theory) over a broad range of cascade parameter values. This result confirms earlier conclusions based on the finite cascade analysis[1,8,10] and Kurosaka's low-frequency analysis,[6] and indicates the important influence that cascading has on airfoil response. Although numerical examples have been restricted to rigid, single-degree-of-freedom blade motions, the present solution is clearly valid for more complicated motions including elastic deformations of the blades. Further work is desirable to prove convergence of the infinite series which occur in the present solution procedure, and to improve upon the wake iteration procedure outlined in Eqs. (31) and (32).

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